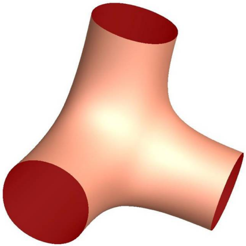
The Statistics of Intersections of Curves on Surfaces

Rachel Zhang
Mentor: Professor Moira Chas at SUNY

PRIMES Conference
May 16, 2015

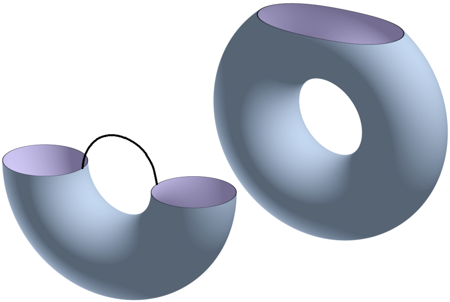
Surfaces



Pair of Pants

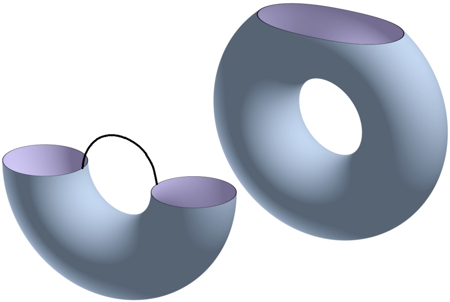
Torus with One Boundary

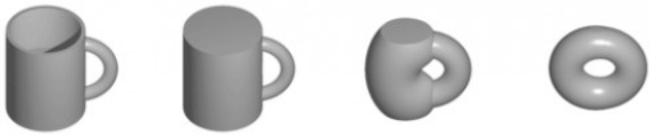
Surfaces
Deformation



Surfaces

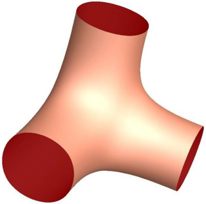
Deformation

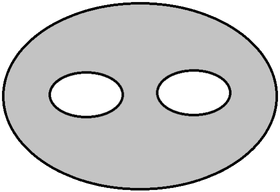




Surfaces and Words

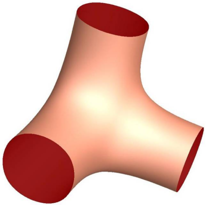
Surfaces and Words
Pair of Pants

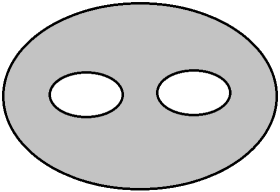


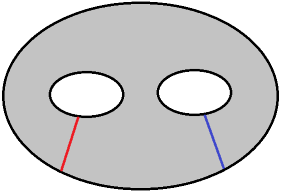


Surfaces and Words

Pair of Pants







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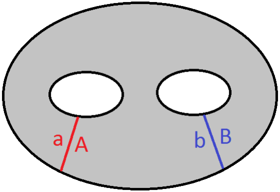

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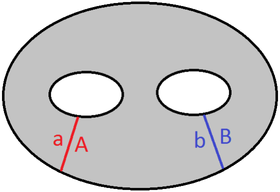

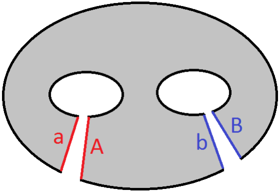

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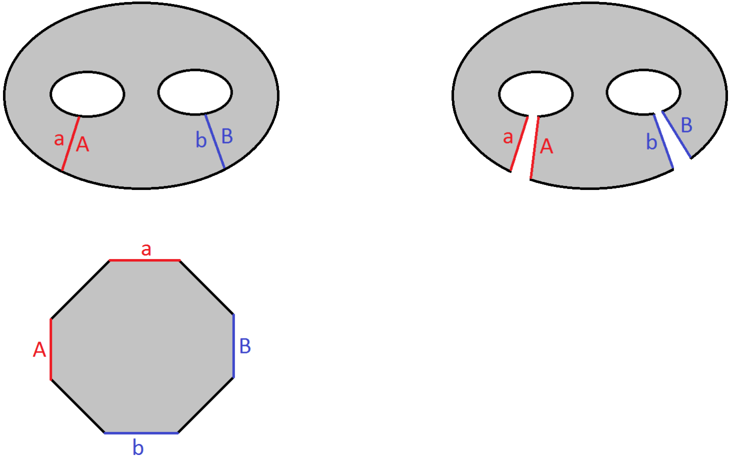

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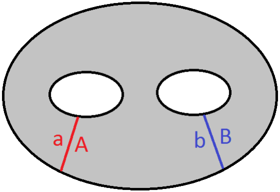
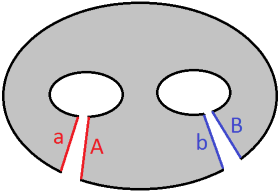
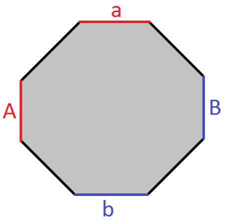






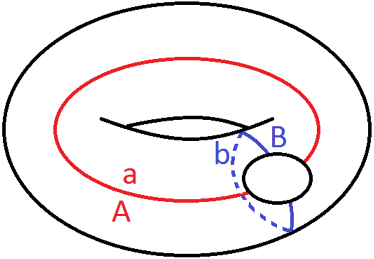
Surface Word = aAbB

Surfaces and Words

Torus with One Boundary

Surfaces and Words

Torus with One Boundary



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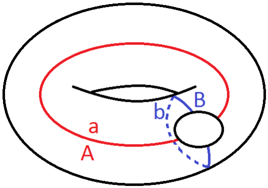

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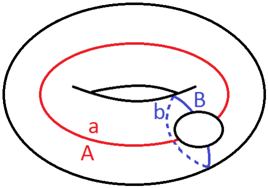
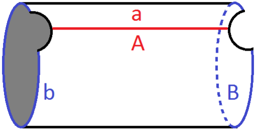
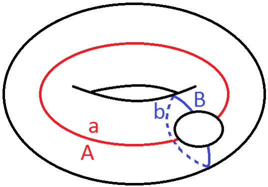
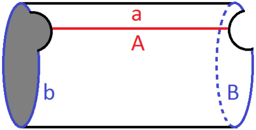
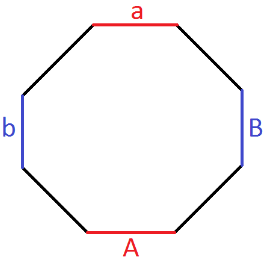




Surfaces and Words
Torus with One Boundary

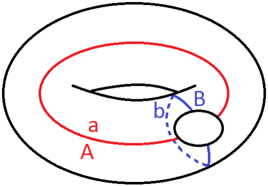


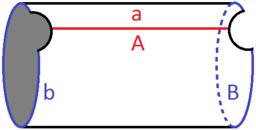


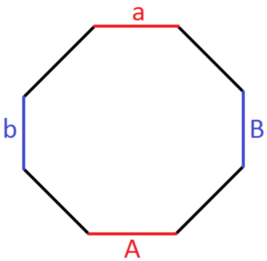


Surfaces and Words

Torus with One Boundary

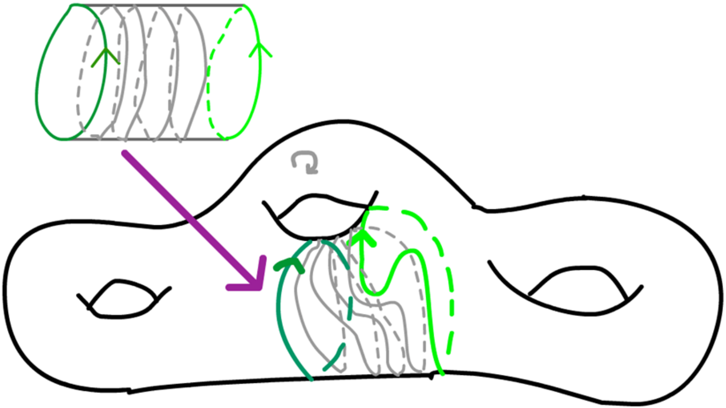


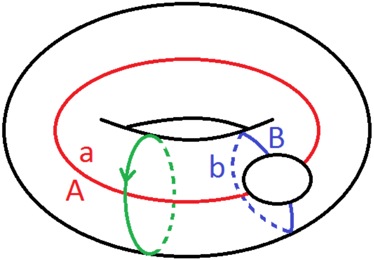




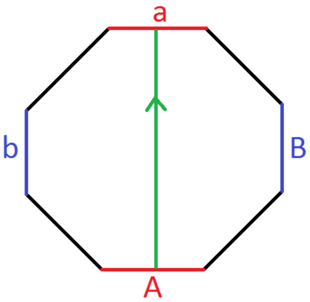
Surface Word = abAB

Two curves are **homotopic** if one can be deformed into the other.



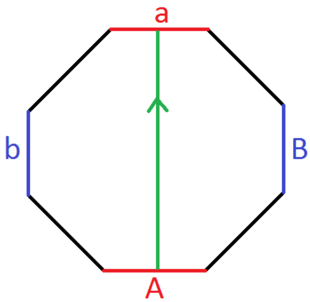


B
a
b
A



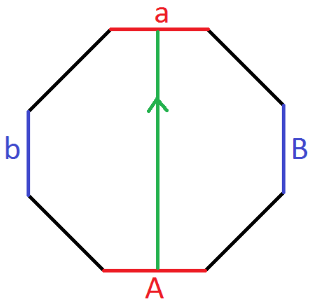
a
b
B
A

Curves on a Surface
Curve Words and Length

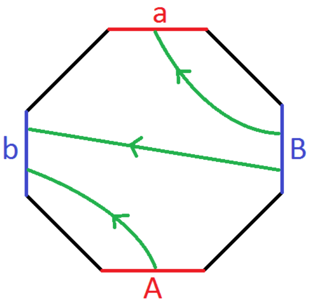


Curve Word $= a$
Curve Length $= 1$

Curves on a Surface
Curve Words and Length



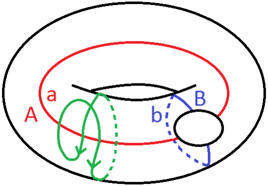
Curve Word $= a$
Curve Length $= 1$

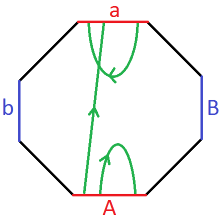


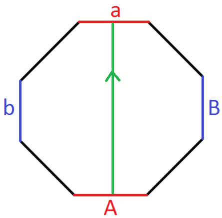
Curve Word $= abb$
Curve Length $= 3$

Curves to Study

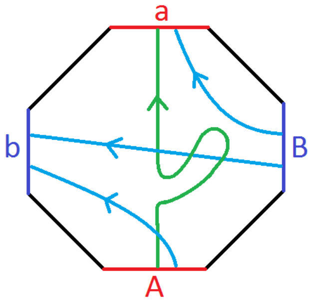
- cyclicly reduced words



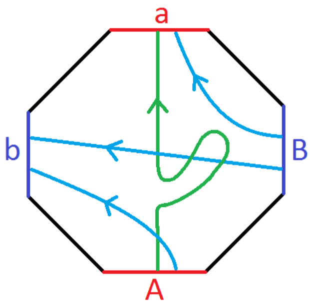


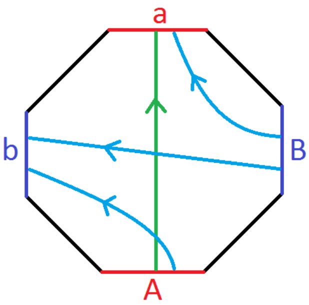


Intersections of Curves

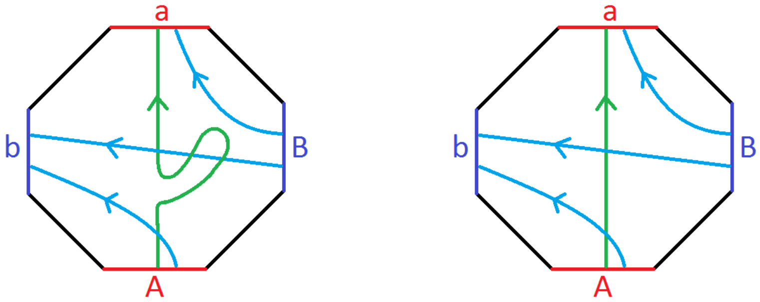


Intersections of Curves





Intersections of Curves



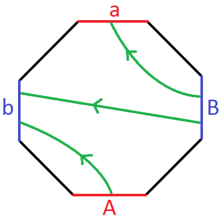
$$i(a, abb) = 2$$

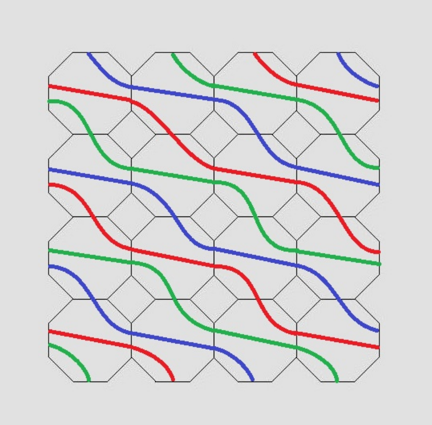
Distribution of Intersections

- Fix a curve ω on a surface S.
- Let n be a positive integer.
- We want to study the distribution of the number of intersections of curves of length n with ω.

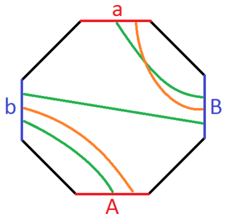
Extended Planar Model

Curve abb on Torus abAB



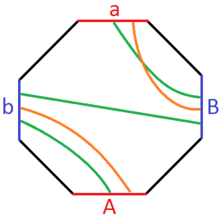


Linked Pairs

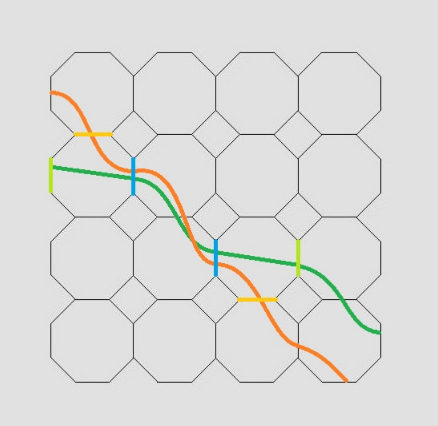


ab and abb on abAB

Linked Pairs



ab and abb on abAB



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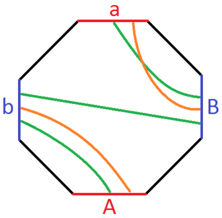
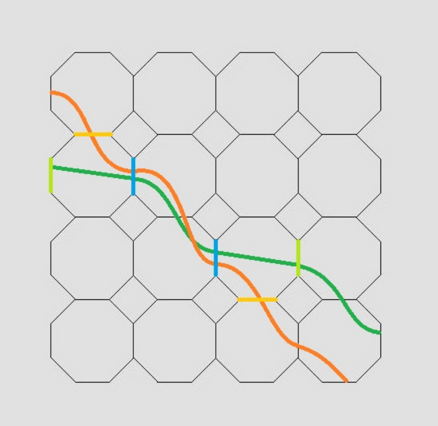


ab and abb on abAB



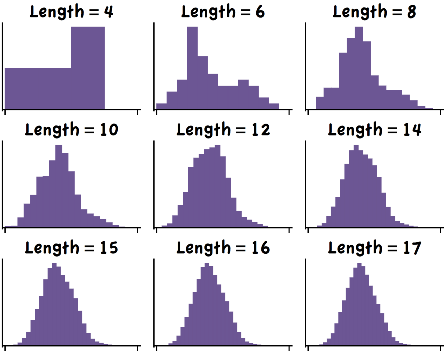
Linked pair = $(ababa, bbabb)$

Mean Number of Intersections

- After determining the complete set of all linked pairs, we can find the probability of each occuring at a specific location in a curve word.
- For example, $P(ababa) = \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$.
- Summing all these probabilities and multiplying by n gives the expected number of intersections of ω and a curve of length n.

Conjecture

The limiting distribution of the number of intersections of ω with curves of length n approaches a Gaussian distribution when normalized.



What's next?

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- standard deviation of distribution of $i(\omega, c)$

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- standard deviation of distribution of $i(\omega, c)$
- relationship between self intersection of ω and the distribution of $i(\omega, c)$

Acknowledgments

- Thanks to my mentor Professor Moira Chas for giving me this project and working with me every week.
- Thanks also to PRIMES and all people working with it for providing me with this experience.